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Bounds on the growth of the magnetic energy for the Hall kinematic dynamo equation

Manuel Núñez

Departamento de Análisis Matemático Universidad de Valladolid 47005 Valladolid, Spain

E-mail: mnjmhd@am.uva.es

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Abstract
While the magnetic induction equation in plasmas, governing kinematic dynamos, is a linear one admitting exponential growth of the magnetic energy for certain velocity fields, the addition of the Hall term turns it into a nonlinear parabolic equation. Local existence of solutions may be proved, but in contrast with the magnetohydrodynamics case, for a number of boundary conditions the magnetic energy grows at most linearly in time for stationary velocity fields, and like the square of the time in the general case. It appears that the Hall effect enhances diffusivity in some way to compensate for the positive contribution of the transport of the magnetic field by the flow occurring in fast dynamos.

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1. Introduction

The classical magnetohydrodynamics (MHD) model treats the plasma as a charged fluid formed by a single species of particles. While reasonably accurate in many large-scale phenomena, the limitations of the model have become clear in recent studies of magnetic reconnection [1–4]. It turns out that the separation of ions and electrons is an essential feature in fast reconnection, so the MHD system should be replaced in these situations by a more precise description. Two-fluid (or electron) MHD is the next simplest model, and a very satisfactory one when one does not need to discriminate between different species of ions. The basic equations may be found in several classic books and review papers, such as [5–7]. After some manipulation, they reduce in the incompressible case to

$$\frac{\partial \mathbf{v}}{\partial t} = \nu \Delta \mathbf{v} - \mathbf{v} \cdot \nabla \mathbf{v} + \mathbf{B} \cdot \nabla \mathbf{B} - \nabla \left(p + \frac{1}{2} B^2 \right) + \mathbf{f} \tag{1}$$

$$\frac{m}{e^2 n} \frac{\partial \nabla \times \mathbf{J}}{\partial t} + \frac{\partial \mathbf{B}}{\partial t} = \eta \Delta \mathbf{B} + \nabla \times (\mathbf{v} \times \mathbf{B}) - \frac{1}{en} \nabla \times (\mathbf{J} \times \mathbf{B}), \tag{2}$$

where the meaning of the magnitudes is as follows: $\mathbf{v}$ is the ions' velocity, $\mathbf{B}$ is the magnetic field, $\mathbf{J} = \nabla \times \mathbf{B}$ is the current density, p is the pressure, $\mathbf{f}$ is an arbitrary force on the momentum equation, ν is the viscosity, η is the resistivity, m is the electron mass, e its charge, and n is the number density. The value $h = 1/en$ is the Hall coefficient. Since m is very small, we will consider it null, and call the remaining equations the Hall system. The whole system (1), (2) turns out to be mathematically consistent, and local existence of solutions may be proved for a variety of boundary conditions [8], but the Hall system is more commonly used in numerical experiments and a theoretical study of its features is advisable. Mostly, we will concentrate on the induction equation (2) and take the velocity $\mathbf{v}$ as a datum uninfluenced by the magnetic field. This procedure has a long history: in dynamo theory it is known as the kinematic dynamo, and it has been studied in depth in order to analyse which kinds of flows yield exponential growth of the magnetic energy. Fast dynamos, where exponential growth occurs even in the limit of zero resistivity, are particularly interesting [9]. Any kinematic approach is valid only for a finite time, before the Lorentz force acts effectively upon the velocity. Kinematic MHD has an essential feature lacking in the Hall version: the induction equation is linear with respect to the magnetic field. Thus its mathematical study, if not straightforward, is predictable in a number of ways. For instance, when the velocity is smooth and bounded, unique solutions exist for all time and one can expect at most exponential growth. That this type of growth really exists for certain flows is an important result of kinematic dynamo theory. We intend to analyse the analogous problem for the (nonlinear) incompressible Hall induction equation and will find that in contrast to the MHD case, and for a number of boundary conditions (including Dirichlet and periodic ones) the Hall equation does not admit exponentially increasing solutions for all time: the rate of growth of $\|\mathbf{B}\|_2$ is at most linear in time. It is curious that the nonlinearity of this equation, instead of allowing the possibility of blow-up like many others, tends to dampen the possible growth of magnetic energy. Since the study of fast dynamos was originally motivated by certain astrophysical phenomena where rapid magnetic field growth is apparent (notably in magnetic stars, such as the Sun) and fast magnetic reconnection, which as stated needs the Hall effect, also occurs in the same objects, it seems reasonable to consider the same set of equations for both instances.

Finally, note that unlike classical MHD, the Hall system cannot be defined in less than three spatial dimensions. However, the magnitudes themselves may be allowed to depend only on two spatial coordinates: in fact, most theoretical studies so far have been performed in this case (see, e.g., [10]).

2. Local existence of solutions

In order to pose properly the problem, we need to fix the boundary conditions. Except for the periodic case, Ω will be a smooth three-dimensional bounded domain. We will consider the following classical conditions:

(1) *Periodic problem.* Ω is a box $[0, L]^3$, $\mathbf{B}$ and $\mathbf{v}$ are periodic at opposite sides of it, and

$$\int_\Omega \mathbf{B}\, \mathrm{d}V = \int_\Omega \mathbf{v}\, \mathrm{d}V = 0.$$

(2) *Dirichlet problem.* $\mathbf{B}|_{\partial\Omega} = \mathbf{v}|_{\partial\Omega} = \mathbf{0}$.
(3) *Perfect conductor.* $\mathbf{v}|_{\partial\Omega} = \mathbf{0}$, $\mathbf{B}\cdot\mathbf{n}|_{\partial\Omega} = 0$, $\mathbf{J}\times\mathbf{n}|_{\partial\Omega} = \mathbf{0}$.

Many other conditions are possible, provided the highest order operator is an elliptic one. Additionally, we must impose $\nabla \cdot \mathbf{B} = 0$ plus the incompressibility condition $\nabla \cdot \mathbf{v} = 0$. Therefore we have

$$-h\nabla \times (\mathbf{J} \times \mathbf{B}) = h\mathbf{J} \cdot \nabla\mathbf{B} - h\mathbf{B} \cdot \nabla\mathbf{J}.$$

Thus the highest (second-)order differential operator in (2) is

$$\mathbf{B} \to \eta\Delta\mathbf{B} - h\mathbf{B} \cdot \nabla\mathbf{J}, \tag{3}$$

which may be written following the convention of summing in repeated indices as

$$\mathbf{B} \to \begin{pmatrix} \eta\partial_{i,i} & B_i\partial_{i,3} & -B_i\partial_{i,2} \\ -B_i\partial_{i,3} & \eta\partial_{i,i} & B_i\partial_{i,1} \\ B_i\partial_{i,2} & -B_i\partial_{i,1} & \eta\partial_{i,i} \end{pmatrix} \mathbf{B}, \tag{4}$$

and for any fixed function $\mathbf{B}_*$ and real vector $\mathbf{x}$, the matrix

$$M_{\mathbf{B}_*} = \begin{pmatrix} \eta x^2 & (\mathbf{B}_* \cdot \mathbf{x})x_3 & -(\mathbf{B}_* \cdot \mathbf{x})x_2 \\ -(\mathbf{B}_* \cdot \mathbf{x})x_3 & \eta x^2 & (\mathbf{B}_* \cdot \mathbf{x})x_1 \\ (\mathbf{B}_* \cdot \mathbf{x})x_2 & -(\mathbf{B}_* \cdot \mathbf{x})x_1 & \eta x^2 \end{pmatrix}, \tag{5}$$

is positive definite uniformly in $\mathbf{B}_*$: for any *real* vector $\mathbf{y}$,

$$M_{\mathbf{B}_*}(\mathbf{y}) \cdot \mathbf{y} = \eta x^2 |\mathbf{y}|^2. \tag{6}$$

An analogous consideration applies to the whole Hall MHD system. In that case, the variables to be considered are the two fields $(\mathbf{v}, \mathbf{B})$, but the highest order operator upon the velocity is simply the Laplacian.

Hence the operator

$$L_{\mathbf{B}_*} : \mathbf{B} \to \eta\Delta\mathbf{B} - h\mathbf{B}_* \cdot \nabla\mathbf{J}, \tag{7}$$

is an elliptic one. Take any set of boundary conditions such that the subspace D_L of the Sobolev space $H^2(\Omega)$ formed by the solenoidal functions satisfying them is such that

$$L_{\mathbf{B}_*} : D_L \to L^2(\Omega)^3 \tag{8}$$

is a bijection for any $\mathbf{B}_* \in \mathcal{C}^1(\bar{\Omega})^3$. This means that the equation $L_{\mathbf{B}_*}\mathbf{F} = \mathbf{f}$ has a unique solution in D_L for any $\mathbf{f} \in L^2(\Omega)^3$; this holds for all the previously stated conditions and many others. Assume also that $\mathbf{v} \in \mathcal{C}^2(\bar{\Omega} \times [0, \infty))^3$ (although this condition could be relaxed). Then the Hall induction equation (2), for any solenoidal initial condition $\mathbf{B}(0) \in \mathcal{C}^1(\Omega)^3$ has a unique solution for some interval $t \in [0, t_0)$; moreover, $\mathbf{B}(t) \in D_L$ for all $t > 0$, which means that $\mathbf{B}$ satisfies the boundary conditions. The result is a consequence of general theorems on nonlinear parabolic equations; see, e.g., [11], pp 169–81. It may be applied as well to the whole Hall MHD system.

3. Bounds on the magnetic energy

We will assume that the boundary conditions satisfy two properties. The first one is commonly used for energy inequalities and it states that there is no seepage of magnetic energy through the boundary of Ω:

$$\frac{1}{2}\int_{\partial\Omega} \frac{\partial B^2}{\partial n}\,\mathrm{d}\sigma = \int_{\partial\Omega} \mathbf{B} \cdot \frac{\partial\mathbf{B}}{\partial n}\,\mathrm{d}\sigma = 0. \tag{9}$$

This equality holds in particular for boundary conditions (1)–(3) above. The second condition is directed to the Hall term and is less usual:

$$\int_{\partial\Omega} ((\mathbf{J} \times \mathbf{B}) \times \mathbf{B}) \cdot \mathbf{n}\,\mathrm{d}\sigma = 0. \tag{10}$$

This happens, for example, if $\mathbf{B}\times\mathbf{n}=\mathbf{0}$, or $\mathbf{J}\cdot\mathbf{n}=\mathbf{B}\cdot\mathbf{n}=0$. Thus it always holds for the Dirichlet condition (2). The integral also vanishes in the periodic case, provided $\mathbf{J}$ is also periodic: in general only $\mathbf{J}\cdot\mathbf{n}$ is periodic. For the perfect conductor case, (10) holds, for example, if the current density vanishes at the boundary.

We will also assume

$$\int_{\partial\Omega}(\mathbf{B}\cdot\mathbf{n})(\mathbf{B}\cdot\mathbf{v})\,\mathrm{d}\sigma=0,$$

which holds for all cases (1)–(3), and also when the velocity satisfies a no-slip condition: $\mathbf{v}|_{\partial\Omega}=\mathbf{0}$.

When $\mathbf{B}$ is smooth enough for Gauss' theorem to apply (certainly in the previous interval of existence, where $\mathbf{B}\in H^2(\Omega)$),

$$\frac{1}{2}\frac{\partial}{\partial t}\int_\Omega B^2\,\mathrm{d}V=\eta\int_\Omega\Delta\mathbf{B}\cdot\mathbf{B}\,\mathrm{d}V-\frac{1}{2}\int_\Omega\mathbf{v}\cdot\nabla B^2\,\mathrm{d}V+\int_\Omega\mathbf{B}\cdot\nabla\mathbf{v}\cdot\mathbf{B}\,\mathrm{d}V-h\int_\Omega\nabla\times(\mathbf{J}\times\mathbf{B})\cdot\mathbf{B}\,\mathrm{d}V. \tag{11}$$

Now

$$\int_\Omega\Delta\mathbf{B}\cdot\mathbf{B}\,\mathrm{d}V=\int_{\partial\Omega}\mathbf{B}\cdot\frac{\partial\mathbf{B}}{\partial n}\,\mathrm{d}\sigma-\int_\Omega|\nabla\mathbf{B}|^2\,\mathrm{d}V=-\int_\Omega|\nabla\mathbf{B}|^2\,\mathrm{d}V\leqslant-\int_\Omega J^2\,\mathrm{d}V \tag{12}$$

$$\int_\Omega\mathbf{v}\cdot\nabla B^2\,\mathrm{d}V=\int_{\partial\Omega}B^2(\mathbf{v}\cdot\mathbf{n})\,\mathrm{d}\sigma=0, \tag{13}$$

$$\int_\Omega\nabla\times(\mathbf{J}\times\mathbf{B})\cdot\mathbf{B}\,\mathrm{d}V=\int_{\partial\Omega}((\mathbf{J}\times\mathbf{B})\times\mathbf{B})\cdot\mathbf{n}\,\mathrm{d}\sigma+\int_\Omega(\mathbf{J}\times\mathbf{B})\cdot\mathbf{J}\,\mathrm{d}V=0. \tag{14}$$

Therefore the classical energy inequality holds for the Hall induction equation,

$$\frac{1}{2}\frac{\partial}{\partial t}\|\mathbf{B}\|_2^2\leqslant-\eta\|\mathbf{J}\|_2^2+\int_\Omega\mathbf{B}\cdot\nabla\mathbf{v}\cdot\mathbf{B}\,\mathrm{d}V. \tag{15}$$

Let us consider now a stream function $\mathbf{w}$, i.e., a function such that $\nabla\times\mathbf{w}=\mathbf{v}$. Since $\mathbf{v}\cdot\mathbf{n}|_{\partial\Omega}=0$, we may take $\mathbf{w}$ such that $\mathbf{w}\times\mathbf{n}|_{\partial\Omega}=\mathbf{0}$; $\mathbf{w}$ also satisfies that its H^1-norm is bounded by a constant, depending only on the domain Ω, times the L^2-norm of $\mathbf{v}$ (see, e.g., [12]). Then, multiplying the induction equation by $\mathbf{w}$,

$$\int_\Omega\frac{\partial\mathbf{B}}{\partial t}\cdot\mathbf{w}\,\mathrm{d}V=-\eta\int_\Omega(\nabla\times\mathbf{J})\cdot\mathbf{w}\,\mathrm{d}V+\int_\Omega(\nabla\times(\mathbf{v}\times\mathbf{B}))\cdot\mathbf{w}\,\mathrm{d}V-h\int_\Omega(\nabla\times(\mathbf{J}\times\mathbf{B}))\cdot\mathbf{w}\,\mathrm{d}V. \tag{16}$$

We have

$$\int_\Omega(\nabla\times\mathbf{J})\cdot\mathbf{w}\,\mathrm{d}V=\int_{\partial\Omega}(\mathbf{J}\times\mathbf{w})\cdot\mathbf{n}\,\mathrm{d}\sigma+\int_\Omega\mathbf{J}\cdot(\nabla\times\mathbf{w})\,\mathrm{d}V=\int_\Omega\mathbf{J}\cdot\mathbf{v}\,\mathrm{d}V. \tag{17}$$

Also

$$\int_\Omega(\nabla\times(\mathbf{v}\times\mathbf{B}))\cdot\mathbf{w}\,\mathrm{d}V=\int_{\partial\Omega}((\mathbf{v}\times\mathbf{B})\times\mathbf{w})\cdot\mathbf{n}\,\mathrm{d}\sigma+\int_\Omega(\mathbf{v}\times\mathbf{B})\cdot\mathbf{v}\,\mathrm{d}V=0. \tag{18}$$

Finally,

$$\int_\Omega(\nabla\times(\mathbf{J}\times\mathbf{B}))\cdot\mathbf{w}\,\mathrm{d}V=\int_\Omega(\nabla\times(\mathbf{B}\cdot\nabla\mathbf{B}))\cdot\mathbf{w}\,\mathrm{d}V=\int_{\partial\Omega}((\mathbf{B}\cdot\nabla\mathbf{B})\times\mathbf{w})\cdot\mathbf{n}\,\mathrm{d}\sigma+\int_\Omega\mathbf{B}\cdot\nabla\mathbf{B}\cdot\mathbf{v}\,\mathrm{d}V. \tag{19}$$

The boundary integral vanishes, and

$$\int_\Omega \mathbf{B}\cdot\nabla\mathbf{B}\cdot\mathbf{v}\,\mathrm{d}V = \int_\Omega \mathbf{B}\cdot\nabla(\mathbf{B}\cdot\mathbf{v})\,\mathrm{d}V - \int_\Omega \mathbf{B}\cdot\nabla\mathbf{v}\cdot\mathbf{B}\,\mathrm{d}V = \int_{\partial\Omega}(\mathbf{B}\cdot\mathbf{n})(\mathbf{B}\cdot\mathbf{v})\,\mathrm{d}\sigma - \int_\Omega \mathbf{B}\cdot\nabla\mathbf{v}\cdot\mathbf{B}\,\mathrm{d}V. \quad (20)$$

Therefore

$$\int_\Omega \frac{\partial\mathbf{B}}{\partial t}\cdot\mathbf{w}\,\mathrm{d}V = -\eta\int_\Omega \mathbf{J}\cdot\mathbf{v}\,\mathrm{d}V + h\int_\Omega \mathbf{B}\cdot\nabla\mathbf{v}\cdot\mathbf{B}\,\mathrm{d}V. \quad (21)$$

By combining (15) and (21) we obtain

$$\frac{h}{2}\frac{\partial}{\partial t}\|\mathbf{B}\|_2^2 - \int_\Omega \frac{\partial\mathbf{B}}{\partial t}\cdot\mathbf{w}\,\mathrm{d}V \leqslant -\eta h\|\mathbf{J}\|_2^2 + \eta\int_\Omega \mathbf{J}\cdot\mathbf{v}\,\mathrm{d}V. \quad (22)$$

Now, since by Young's inequality

$$\int_\Omega \mathbf{J}\cdot\mathbf{v}\,\mathrm{d}V \leqslant h\|\mathbf{J}\|_2^2 + \frac{1}{4h}\|\mathbf{v}\|_2^2, \quad (23)$$

we get

$$\frac{h}{2}\frac{\partial}{\partial t}\|\mathbf{B}\|_2^2 - \int_\Omega \frac{\partial\mathbf{B}}{\partial t}\cdot\mathbf{w}\,\mathrm{d}V \leqslant \frac{\eta}{4h}\|\mathbf{v}\|_2^2. \quad (24)$$

Assume first that $\mathbf{v}$ is stationary; obviously, we may take $\mathbf{w}$ also stationary, so that

$$\int_\Omega \frac{\partial\mathbf{B}}{\partial t}\cdot\mathbf{w}\,\mathrm{d}V = \frac{\partial}{\partial t}\int_\Omega \mathbf{B}\cdot\mathbf{w}\,\mathrm{d}V. \quad (25)$$

By integrating (24) in time, we obtain

$$\frac{h}{2}\|\mathbf{B}(t)\|_2^2 \leqslant \int_\Omega (\mathbf{B}(t)-\mathbf{B}(0))\cdot\mathbf{w}\,\mathrm{d}V + \frac{h}{2}\|\mathbf{B}(0)\|_2^2 + \frac{\eta}{4h}\|\mathbf{v}\|_2^2 t, \quad (26)$$

which implies that

$$\frac{h}{2}\|\mathbf{B}(t)\|_2^2 - \|\mathbf{B}(t)\|_2\|\mathbf{w}\|_2 \leqslant \frac{h}{2}\|\mathbf{B}(0)\|_2^2 + \|\mathbf{B}(0)\|_2\|\mathbf{w}\|_2 + \frac{\eta}{4h}\|\mathbf{v}\|_2^2 t. \quad (27)$$

Therefore

$$\|\mathbf{B}(t)\|_2 \leqslant \frac{1}{h}\|\mathbf{w}\|_2 + \left[\frac{1}{h^2}\|\mathbf{w}\|_2^2 + \|\mathbf{B}(0)\|_2^2 + \frac{2}{h}\|\mathbf{B}(0)\|_2\|\mathbf{w}\|_2 + \frac{\eta}{2h^2}\|\mathbf{v}\|_2^2 t\right]^{1/2}, \quad (28)$$

so that the magnetic energy $\|\mathbf{B}\|_2^2$ grows at most linearly in time.

Velocities constant in time have been used in fast dynamo studies, notably several variants of the ABC flow [13, 14]; however, most studies deal with time-dependent velocities, often periodic in time [15]. We may safely assume that $\|\partial\mathbf{v}/\partial t\|_2$ (and therefore $\|\partial\mathbf{w}/\partial t\|_2$) remain bounded in time. Since

$$\frac{\partial}{\partial t}\int_\Omega \mathbf{B}\cdot\mathbf{w}\,\mathrm{d}V = \int_\Omega \frac{\partial\mathbf{B}}{\partial t}\cdot\mathbf{w}\,\mathrm{d}V + \int_\Omega \mathbf{B}\cdot\frac{\partial\mathbf{w}}{\partial t}\,\mathrm{d}V, \quad (29)$$

from (24) we obtain

$$\frac{\partial}{\partial t}\left(\frac{h}{2}\|\mathbf{B}\|_2^2 - \int_\Omega \mathbf{B}\cdot\mathbf{w}\,\mathrm{d}V\right) \leqslant \frac{\eta}{4h}\|\mathbf{v}\|_2^2 + \|\mathbf{B}\|_2\left\|\frac{\partial\mathbf{w}}{\partial t}\right\|_2. \quad (30)$$

Integrating this inequality in time,

$$\frac{h}{2}\|\mathbf{B}(t)\|_2^2 \leqslant \|\mathbf{B}(t)\|_2\|\mathbf{w}(t)\|_2 + \frac{h}{2}\|\mathbf{B}(0)\|_2^2 + \|\mathbf{B}(0)\|_2\|\mathbf{w}(0)\|_2 + \frac{\eta}{4h}\int_0^t \|\mathbf{v}(s)\|_2^2\,\mathrm{d}s + \int_0^t \|\mathbf{B}(s)\|_2\left\|\frac{\partial\mathbf{w}}{\partial s}(s)\right\|_2\,\mathrm{d}s. \quad (31)$$

To analyse the behaviour of this inequality, let us simplify it by taking a bound M on all the functions which are known to be bounded. We get

$$\|\mathbf{B}(t)\|_2^2 \leqslant M\|\mathbf{B}(t)\|_2 + M + Mt + M\int_0^t \|\mathbf{B}(s)\|_2\,\mathrm{d}s. \tag{32}$$

Although this inequality may be integrated explicitly, it yields an ugly expression too complex to be of much help. It is simpler to study the possible growth of $\|\mathbf{B}\|_2$ by a simple order analysis: if it grows like t^α, the right-hand side grows at most like $t^{\alpha+1}$, whereas the left one behaves like $t^{2\alpha}$. Thus $2\alpha \leqslant \alpha + 1$, namely $\alpha \leqslant 1$. Hence the magnetic energy $\|\mathbf{B}\|_2^2$ grows at most like t^2.

4. Conclusions

The magnetic induction equation, taking into account the Hall effect but considering the plasma velocity as a datum, is a nonlinear parabolic one governing Hall kinematic dynamos. Existence of solutions may be proved for a finite time for smooth enough initial values of the magnetic field and for a variety of boundary conditions; incidently, the same argument may be applied to the full Hall MHD system. The kinematic dynamo problem in classical MHD yields a linear equation in which the magnetic field grows at most exponentially; this possibility actually occurs for a number of well-known velocity fields. Rather surprisingly, the Hall term actually inhibits the magnetic growth in the sense that the magnetic energy grows at most linearly in time for stationary velocities and at most quadratically for general ones, provided the L^2-norm of the time derivative of the velocity remains bounded, as it happens in all known examples. Apparently, the Hall effect precludes the transport of the magnetic field by the flow, plus the positive folding, present in all successful instances of fast dynamos.

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